

Exercise 20

Differentiate the function.

$$H(z) = \ln \sqrt{\frac{a^2 - z^2}{a^2 + z^2}}$$

Solution

Take the derivative of the function.

$$\begin{aligned} H'(z) &= \frac{d}{dz} \left(\ln \sqrt{\frac{a^2 - z^2}{a^2 + z^2}} \right) \\ &= \frac{d}{dz} \left[\frac{1}{2} \ln \left(\frac{a^2 - z^2}{a^2 + z^2} \right) \right] \\ &= \frac{1}{2} \frac{d}{dz} [\ln(a^2 - z^2) - \ln(a^2 + z^2)] \\ &= \frac{1}{2} \left[\frac{d}{dz} \ln(a^2 - z^2) - \frac{d}{dz} \ln(a^2 + z^2) \right] \\ &= \frac{1}{2} \left[\frac{1}{a^2 - z^2} \cdot \frac{d}{dz}(a^2 - z^2) - \frac{1}{a^2 + z^2} \cdot \frac{d}{dz}(a^2 + z^2) \right] \\ &= \frac{1}{2} \left[\frac{1}{a^2 - z^2} \cdot (-2z) - \frac{1}{a^2 + z^2} \cdot (2z) \right] \\ &= -\frac{z}{a^2 - z^2} - \frac{z}{a^2 + z^2} \\ &= \frac{-z(a^2 + z^2) - z(a^2 - z^2)}{(a^2 - z^2)(a^2 + z^2)} \\ &= \frac{-2a^2z}{a^4 - z^4} \\ &= \frac{2a^2z}{z^4 - a^4} \end{aligned}$$